Estimated candy Maximization: A Physical Approach

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1 Introduction

The problem that kids face every year is that despite all the hype, a clear strategy for maximizing candy is never devised, mainly because it's practically impossible. The purpose of this article is to estimate an upper bound for the amount of candy which a suburban kid can hope to collect.

2 Assumptions of the Problem

The influx of candy is not a constant rate because as more candy fills the bag, the trick or treater runs slower. For simplicity we will do calculations assuming this is constant to figure out how much velocity changes and then move to correct our assumption. Assume that all the candy is homogeneous in weight and thus each candy bar is 17 grams in weight (modeled in terms of fun-sized Milky Way bars). On average, each house will give about one piece ignoring the chances that a bowl is found. The choice of only choosing to allow each house to give one piece of candy in mass takes into the fact that some houses may end in a bust, and some houses may give more than one piece. Let the amount of candy's mass dropping into the bag be Φ. This is found by $\frac{M_c}{time}$. Notice that time is distance between houses over speed traveling.

3 Momentum Analysis

The small mass dm changes the momentum of the trick or treater of we assume that his motion is continuous. Because the mass is simply dropping in the the bag, no horizontal force is imparted on the boy but mass slows him down. Conservation of momentum tells us $mv = (dm + m)(v - dv)$ in the preceding calculation we will drop the second order $dmdv$ term and have M be the mass of the trick or treater.

$$dmv = mdv$$

$$\int_M^m \frac{dm}{m} = \int_v^{v_f} \frac{dv}{v} \Rightarrow \ln\left(\frac{m}{M}\right) = \ln\left(\frac{v}{v_f}\right)$$

$$\Rightarrow m = \frac{Mv}{v_f}$$

Notice that as the influx of candy goes to infinity (kid gets infinite candy), the final speed of our trick or treater goes to zero as it should.

But how does the influx of candy affect the speed of the trick or treater? Mass enters at a rate of Φ thus

$$\frac{dm}{dt} = \Phi \tag{1}$$

We treat v in the above equation as that was a given constant and the variable subject to change is how fast the final velocity is (v_f). We substitute in m and set up the differential equation

$$-\int_v^{v(t)} \frac{dv_f}{v_f^2} = \int_0^t \frac{\Phi dt}{Mv} \tag{2}$$

$$\left(\frac{1}{v(t)} - \frac{1}{v}\right) = \frac{\Phi t}{Mv} \Rightarrow v(t) = \frac{Mv}{M + \Phi t} \tag{3}$$

Hold it right there! You should not be satisfied with the above because we really should write Φ as $d\Phi$.

4 Estimating the Maximum Bound

The classic trick or treating times usually last for about two hours, so we will use that as the maximum allotted time. In the previous section we determined how the velocity changes with the influx of candy, but our problem is even more complicated because Φ is also decreasing proportionally as velocity decreases. We can set up our differential equations as follows (D is the distance between houses).

$$\frac{d\Phi}{dt} = \frac{M_c(\frac{dv}{dt})}{D} \tag{4}$$

$$\frac{dv}{dt} = \frac{Mv}{M + \frac{d\Phi}{dt}t} \tag{5}$$

Combine equations (6) with (7) we have a quadratic in terms of $\frac{d\Phi}{dt}$ which we will solve using the quadratic formula and keeping the positive root.

$$\frac{d\Phi}{dt} = \frac{M_c\left(\frac{Mv}{M+\frac{d\Phi}{dt}t}\right)}{D} \Rightarrow \qquad Dt\left(\frac{d\Phi}{dt}\right)^2 + DM\left(\frac{d\Phi}{dt}\right) - M_cMv = 0 \tag{6}$$

$$\frac{d\Phi}{dt} = \frac{-DM + \sqrt{(DM)^2 + 4DtM_cMv}}{2Dt} \tag{7}$$

$$\int d\Phi = \int_0^t \frac{-DM + \sqrt{(DM)^2 + 4DtM_cMv}}{2Dt}dt \tag{8}$$

Obviously when we integral Φ we get the mass so we separate variables and then use Wolfram Alpha to help evaluate the integral on the RHS.

So that we can do this numerically, suppose our trick or treater is 50 kg, ran in the beginning of $v = 2.5\frac{m}{s}$ and that the mass of the candy was 0.017 kg. After subbing all the variables we get that the total mass is an astonishing 25.015 kg worth of candy.

Remark. *This is actually quite reasonable given the assumptions we made in the beginning. We assumed that the number of houses was sufficient and that the distance between each house is* 10 *meters. Doing the calculation without any differential equations and assuming that running speed nor influx rate changes.*

$$\frac{7200(2.5)}{10} = 1800 \Rightarrow 1800 \cdot 0.017 = 30.6kg \tag{9}$$