Forces

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Leave all answers in a form with only variables given and any constants needed.

1. A rectangular board rests on top of a disk which is free to roll. The disk is moving with velocity v. How fast is the board moving relative to the ground if there is no slipping anywhere?

2. This is a classic problem. Patrick and Matthew are playing with a yo-yo which is constructed from a disk of radius r and two larger disks of radius R. There is a string that wraps around the small cylinder and it is pulled with a force T at some angle θ to the horizontal. Patrick claims that not matter how the string is pulled, the cylinder must move the opposite direction, but he's super bad at physics. Show that Patrick is wrong. Find the critical angle for which Patrick is right.

3. A bead rests at the top of a fixed frictionless semicircular hoop of radius r that lies in a vertical plane. The bead is given a tiny push so that it slides down and around the hoop. Consider the horizontal component of the force from the hoop on the bead. At what points on the hoop does this component achieve a local maximum or minimum?

4. Patrick launched with some initial velocity v_0 from an initial position of zero. While in the air, Patrick experiences a force of air resistance $F(v) = -bv^2$. Give the function of Patrick's position, $x(t)$.

5. Matthew and Jeremy each of mass m hate getting dirty so Patrick dangles them by a spring with spring constant k and length l where both are at the bottom. Patrick gets a brilliant idea and constructs another system which involves two springs each of constant k and lengths $l/2$ and places his two friends at the end of each spring. Why is Patrick not very intelligent for devising this plan. Derive the formula for springs in series.

6. Patrick is pretty fat, model him as a point particle. One of end a string is tried to the vertex of an upright cone with half angle θ. The other end of the string is tied to Patrick.

   - Find the tension in the string assuming Patrick is given a kick by Matthew and moves tangentially with a speed of v.
   - Find the force which the cone applies on Patrick as he is rotating but still remaining in the surface of the cone.
   - Find the speed v such that Patrick will not be touching the surface of the cone.

7. Patrick actually does not know how to ride a bike. Suppose that he can. Patrick is of height ℓ rides in a circle of radius R and is leaning at an angle of θ to the vertical. With what angular velocity must Patrick ride to not fall over?

   Remark. *Assume gravitational acceleration is g. Assume that $\ell << R$*

8. Patrick has a cylindrical column of water. If he spins the cylinder giving it some angular velocity ω, what shape will the surface of the water take.

   Remark. *It is sufficient to just consider a cross section of the water, which will be some curve.*

9. ***Challenge:*** Derive the equation for a free hanging chain with its two ends at the same height. The chain is of a homogeneous density ρ. As a hint, the y component of tension, T_y, in the chain is proportional to the slope of the chain.