Gravity Problems Part 2

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Before we begin, there are a few formulas you should know.

$$\frac{\tau^2}{a^3} = \frac{4\pi^2}{GM} \tag{1}$$

$$l = \frac{L}{m} = \frac{|\mathbf{r} \times \mathbf{p}|}{m} = |\mathbf{r} \times \mathbf{v}| = r^2\dot{\theta} \tag{2}$$

$$m\ddot{r} = \frac{ml^2}{r^3} + f(r) \text{ for a central force } f(r) \tag{3}$$

$$f(u^{-1}) = -ml^2u^2\left(\frac{d^2u}{d\theta^2} + u\right) \text{ with } u = \frac{1}{r} \tag{4}$$

Problem 1 Calculate the period of a nearly circular low earth orbit satellite 200 miles above the Earth (Look up numbers as you need).

Problem 2 Given that the orbit of the particle passes through the origin in the path of $r = r_0cos(\theta)$, find the force law.

Problem 3 In this problem, you will determine for what n the power law $f(r) = -cr^n$ admits stable circular orbits. Begin by setting $x = r - a$ where a is the radius of the circular orbit, and rewrite the equation in terms of x and a. Do a taylor series to first order in x, and collect your terms by $\ddot{x}$ and x. The equation you get should look like a special type of equation we have come across before (Hint, think about springs). If we want it to be stable, we need the force to be restoring, in other words, the force has to be in the opposite direction of x. Using $f(r) = -cr^n$, find an inequality for n for the existence of stable orbits.