Useful Formulas for Mechanics

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These formulas are lesser known ones that need to be derived. I assume that everyone knows the basic formulas.

1. 2D motion formulas for range and max height :

$$x_{max} = \frac{v_0^2 \sin(2\theta)}{g}$$
$$y_{max} = \frac{v_0^2 \sin^2(\theta)}{2g}$$

2. Effective spring constant of springs in parallel and series are respectively:

$$k_{eff} = k_1 + k_2 + \cdot + k_n$$
$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + ... + \frac{1}{k_n}$$

3. Consider a circular piece of rope that is rotating with velocity v_0. The tension in the rope is:

$$\lambda v_0^2.$$

This formula is independent of the radius of curvature of the rope and only on the mass density of the rope λ and v_0, and tension is uniform throughout it. This quantity comes up a lot, it is also the amount of force needed to straighten out a heap of rope if you grab one end and pull with velocity v_0.

4. Consider a pulley and two masses m_1 and m_2 on each side connected by a string. The pulley with the masses on it is held up by another piece of string. The tension in the string which keeps the pulley system from moving is:

$$T = \frac{4m_1 m_2}{m_1 + m_2}$$

5. In an elastic collision with a mass m_1 moving a velocity v hitting a stationary mass m_2, the final velocities of the two masses are:

$$v_{m_1} = \frac{(m_1 - m_2)v}{m_1 + m_2}, \quad v_{m_2} = \frac{2m_1 v}{m_1 + m_2}.$$

Notice, the first can be a negative velocity.

6. A rocket starts off with mass M and ejects fuel backwards at a velocity v *relative to the rocket.* After the rocket ejects fuel for some time, it has a new mass m. The new velocity of the rocket is:

$$V_f = v \ln\left(\frac{M}{m}\right)$$

7. For momentum questions with a changing mass (accumulated over a mass density) use :

$$F = \frac{dp}{dt} = m\frac{dv}{dt} + \frac{dm}{dt}v$$

8. Parallel axis theorem:

$$I = I_{cm} + mr^2$$

Here r is the distance between the center of mass and the axis of rotation. Remember that we can subtract moments of inertia if we are cutting something out of something. Like a hole in a disk.

9. Period of a physical pendulum :

$$T = 2\pi\sqrt{\frac{I}{mg\ell}}$$

We have that ℓ is the distance the center of mass of the physical pendulum is away from the pivot. We usually calculate I with the parallel axis theorem. Think of this as the regular point mass pendulum except now we need to take into account torque on a "physical" object.

10. An object with moment of inertia βmr^2 rolling down a plane inclined at θ has linear acceleration:

$$a = \frac{g\sin(\theta)}{1+\beta}$$

11. For an atwood with a massive pulley of moment of inertia βmr^2, the acceleration of the system is:

$$a = \frac{g(M-m)}{\frac{I}{r^2} + M + m}.$$

For M and m are hanging masses and $M > m$.

12. For a Keplerian orbit, the velocity of the object in orbit is:

$$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

where r is the separation distance from the central body M, and a is the semi-major axis of the orbit. Note that $a > 0$ for ellipses, $a = \infty$ for parabolas, $a < 0$ for hyperbolas.

13. Kepler's third law for an object:

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

where a and M are used the same as in 12.

14. Let object m orbit object M in an ellipse such that r is the closet approach and nr is the farthest, perihelion and aphelion respectively. The total energy of the object is:

$$\frac{-GMm}{(n+1)r}$$

15. Consider a steady very "normal" (laminar) flow of water between two pipes of differing areas, velocities are related by:

$$A_1 v_1 = A_2 v_2$$

16. The bouyant force is equal to the weight of the liquid displaced, $\rho V g$:

$$\frac{density\ of\ object}{density\ of\ fluid} = \frac{weight\ of\ object}{weight\ of\ displaced\ fluid}$$