

# Torques and Angular Momentum

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1. Make sure you know the moments of inertia for basic rigid objects. The formula for moment of inertia is  $I = \int r^2 dm$ . The parallel axis theorem says  $I = I_{cm} + mr^2$  for  $r$  is the distance away the axis is from the center. Derive the moment of inertia for the following objects using either the integral, or our clever scaling trick.
  - A rod of length  $\ell$  about the end. Use the integral definition and the parallel axis theorem.
  - A triangle of side length  $\ell$  about the center.
  - A uniform circular disk of radius  $r$  about the center, and about the edge
  - A hoop of radius  $r$  about the center.
  - A hexagon of side length  $\ell$  about the center
2. Patrick has a disk which is of mass  $m$  and radius  $r$ . A stick mass  $M$  and length  $\ell$  is hung from the side of the disk vertically and then let go. Find the initial angular acceleration of the system.
3. A cylinder mass  $m$  and radius  $r$  lies on top of a board of mass  $m$ . The top of the board has a friction coefficient of  $\mu$ , but the bottom is free to slide on the ground without friction. The cylinder is initially given some velocity  $v$  to the right, without slipping. Find the velocity of the cylinder when it starts to roll without slipping.
4. A solid sphere of mass  $m$  and radius  $r$  initially travels at  $v_0$  with slipping. When the ball rolls without slipping, how fast will the ball be going? We know only that friction causes the ball to not slip.
5. Consider a massless stick length  $\ell$  with two rubber masses  $m$  on both ends. Patrick drops the stick so that one of the masses hits the ground first at velocity  $v$ . At that instant, the stick makes an angle  $\theta$  with the horizontal. Find the rotational speed of the stick after the collision.
6. A stick with moment of inertia  $\frac{m\ell^2}{12}$  about the center is initially dropped in the horizontal configuration. It hits ledge when it is  $h$  above the ground, going at velocity  $v$ . If the stick ends up upright after it reaches the ground, making more than one rotation on the way down, find how far from the center of the stick the ledge struck it.