

Special Relativity Part 1

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The following are some of the formulas that were discussed:

$$t = \gamma t', \quad \ell = \frac{\ell'}{\gamma}, \quad v_f = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}, \quad x = \gamma(x' - vt') \quad t = \gamma \left(t' - \frac{vx'}{c^2} \right)$$

1. Patrick, Jeremy, and Arnold race each other in carts all of proper length L . Jeremy travels at $\frac{5c}{7}$, Arnold travels at $\frac{3c}{7}$, and Patrick travels at $\frac{c}{7}$. Eric stands on the ground and sees Jeremy overtake Arnold and then Patrick. Find the amount of time in Eric's frame that it took for Jeremy to go the length of Arnold and Patrick.

Remark. *To overtake means that the front of Jeremy's cart initially lines up with the back of another and he goes until the back of his cart lines up with the front of another.*

2. (*Motivated by Patrick*) Ji and Raymond are separated by a distance of L and each travel at non-relativistic velocity v towards the other. Alex runs at $\frac{3c}{5}$ between the two. Raymond and Ji both carry portals so that once Alex reaches one, he magically gets transported to the other. So, if Alex starts at Raymond, he runs for some time until he arrives at Ji and then transports to Raymond's new position (still headed towards Ji) with no discontinuities.
 - How far does Alex run in his own reference frame?
 - Ethan who is standing still watches as Alex runs. How far does Ethan perceive Alex to have run?
3. Rocket scientist Karina is on her spaceship of proper length L which moves at a speed of $\frac{2c}{5}$. She stands at the back of her spaceship and fires an electron, which travels at $\frac{7c}{10}$, towards the front. Space alien Victoria who is floating in space with no velocity relative to the rocket and watches the electron travel. Find the time it took for the electron to hit the front of the spaceship in Victoria's frame, and also Karina's frame. Find the distance Victoria sees the electron travel.
4. Rona swims at $\frac{4}{5}$ the speed of light in the direction of a river (downstream) which flows at $\frac{c}{5}$. Matthew hears about this and claims that she must be swimming at c because he knows how to add fractions. However, Matthew does not know the first thing about physics so master Vishnu explains relativity to him. How fast does Vishnu say Rona is actually swimming relative to the shore.

Problems 5 and 6 consider two classic paradoxes in special relativity. Discuss the "paradox" in the problem and explain where there is an error in reasoning.

5. Twins Noah and Noah both live on Earth. Noah goes on a rocket to a distant star, turns around, and comes back. Show that the Noah who went on the rocket is younger than the Noah that stayed on Earth at the end of the trip (when both are on Earth again).
6. A train of proper length L drives at v into a tunnel which also has proper length L . Cynthia attaches a radio to the front of the train which will play "Shake it Off" when the front end of the train passes through the far end of the tunnel. Haena also mounts a radio, which will play "Blank Space" by sending a signal to disable Cynthia's radio when the back end of the train passes the close end of the tunnel. Will "Shake it Off" be played? Try to be as quantitative as possible.