Comet and Satellite Intersection Problem

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Central forces such as gravity point inwards towards the source, that is, the force is located at a source with a certain potential $U(r)$. Dealing with these forces is extremely important as they are what cause the planets and comets to dance in their orbits. Recall that

$$\frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + U(r) = E \tag{1}$$

Problem: The Matthew Institute of Technology has just launched a satellite into deep space and the path which it is takes is now nearing a star. The satellite approaches the star with a parabolic trajectory having the star as a focus. The satellite however enters the orbit of a comet which is going in orbit around the star also with a parabolic trajectory having the star as a focus. The time which the satellite spends in the comet's orbit is d days, find the distance from the star in which the trajectories intersect, r_{max}. Assume that the comet and satellite are not affecting each other with their gravitational effects and also assume that the dotted line is the minimum distance that satellite comes to the star.

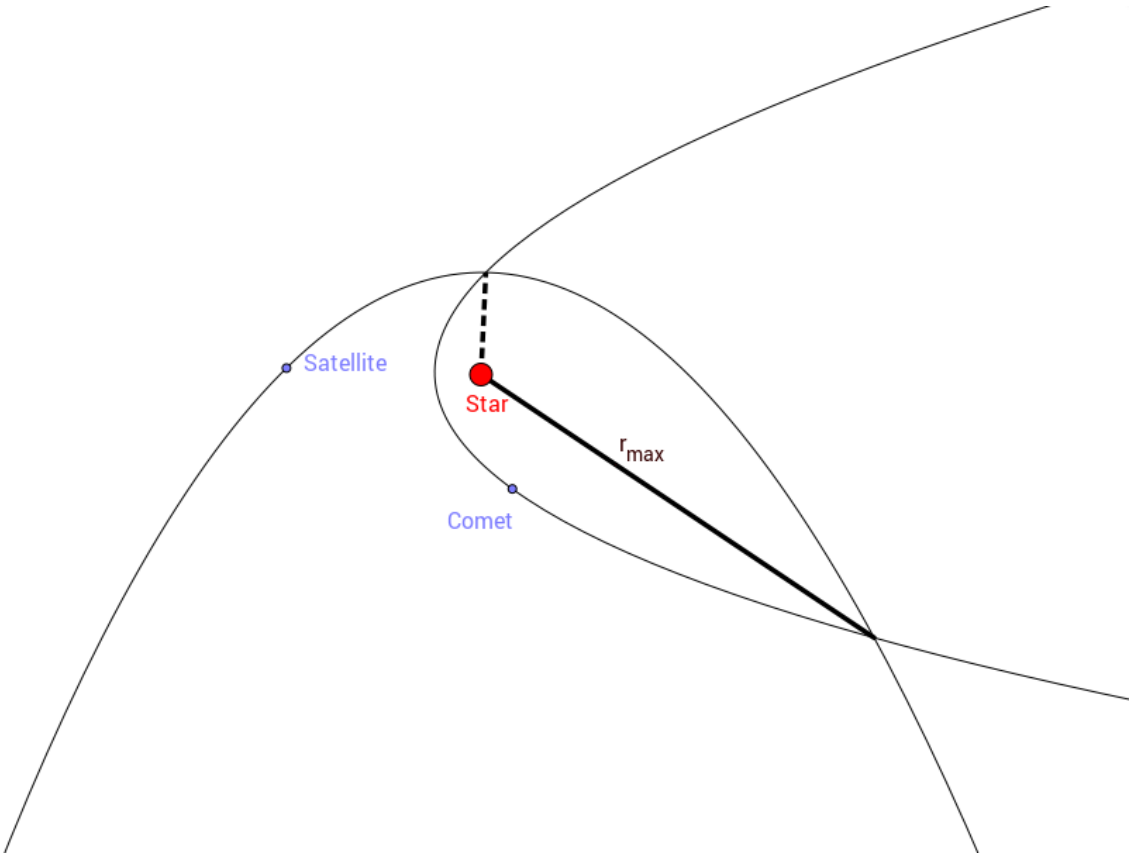

Figure 1: It is worth noting that the the first intersection being also the perihelion is not a coincidence, of course this problem would be even more interesting given a different point of intersection.