Lagrangian Problems Part 2

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1. The force experienced by a charged particle is $F = q\left(\mathbf{E} + \dot{r} \times \mathbf{B}\right)$. The electric and magnetic fields are related to the vector and scalar potential as follows.

$$\mathbf{B} = \nabla \times \mathbf{A}$$
$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}.$$

   Patrick has troubles with physics and maths. Show him that the lagrangian for the charged particle is

$$\mathscr{L} = \frac{1}{2}m\dot{r}^2 - q\phi + q\mathbf{A}\cdot.$$

2. It should be fairly obvious what the path that a charged particle moving into a region with a constant magnetic field $\mathbf{B} = (0, 0, B)$ and zero electric field looks like. However Patrick has no clue of the answer. Prove to him what the path is.

   Remark. *I know everyone is smarter than Patrick. However, just in case the path is a helix.*

3. Mr. Schafer might kill me if he finds out about this problem. Patrick has a ladder length 2ℓ which initially leans against a wall at an angle of θ_0 to the horizontal. Both the wall and the ground are frictionless. Find the angle at which the ladder will lose contact with the wall.

   Remark. *Remember that the ladder has both rotational and translational kinetic energy. You can assume that the ladder loses contact with the wall before it does with the ground, but this should technically be proved.*

4. The scalar and vector potentials in problem 1 are not unique. We claim that

$$\phi \mapsto \phi - \frac{\partial \chi(r,t)}{\partial t}$$
$$\mathbf{A} \mapsto \mathbf{A} + \nabla\chi$$

   will also give the same equation of motion. Notice that $\mathscr{L} \mapsto \mathscr{L} + q\frac{\partial \chi}{\partial t} + q \cdot \nabla\chi$. Show that this new lagrangian is equivalent to the lagrangian derived in problem 1.

   Remark. *This is known as a Gauge Transformation of the lagrangian.*