Lagrangians Problems Part 1

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October 2016

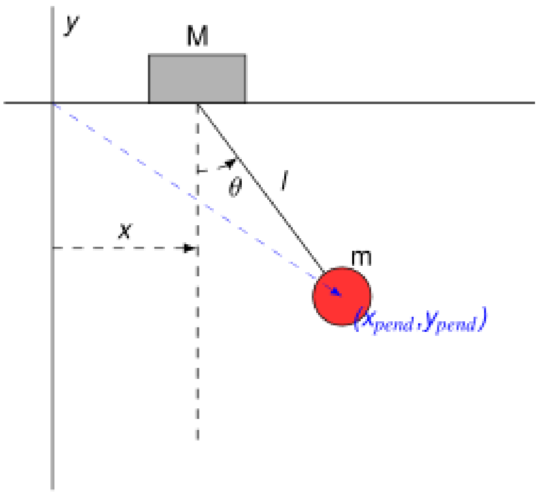
The Lagrangian Formulation is one the most powerful concepts in modern physics. It can be used in fields from Mechanics to General Relativity to Quantum Field Theory. We'll mostly be dealing with Mechanics though. For most Mechanics problems (and all these problems), the Lagrangian is going to be

$$L(q, \dot{q}, t) = T(q, \dot{q}, t) - V(q, \dot{q}, t) \tag{1}$$

where T is Kinetic Energy and V is the Potential Energy. The Euler-Lagrange Equation is

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0 \tag{2}$$

1. Find the acceleration of a block of mass m as it goes down a frictionless ramp of angle θ. Check that is agrees with $gsin(\theta)$.
2. Find the equation of motion of a single fixed pendulum using Newton's Law, and then using Lagrangian Mechanics and check that they agree. Use θ as the angle between the bar of length l and the vertical.
3. You have already found the differential equation for a single fixed pendulum, now find the differential equations of motion for a single pendulum with a movable support. Use x and θ as shown below.



4. Here is a preview of what we (may) do in the future. We define the Hamiltonian as

$$H = \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L \tag{3}$$

- Prove that the Hamiltonian is conserved, or $\frac{dH}{dt} = 0$ assuming that $\frac{\partial L}{\partial t} = 0$. Note: $\frac{dL}{dt} \neq \frac{\partial L}{\partial t}$ (Remember the Multivariate Chain Rule?)
- Let $p_j = \frac{\partial L}{\partial \dot{q}_j}$, which is called the generalized momentum. Show that if $\frac{\partial L}{\partial q_j} = 0$ then $\frac{dp_j}{dt} = 0$

5. **Challenge** : If you are feeling up to it, derive the differential equations of motion for the double pendulum. Use θ_1 and θ_2 as angle between each bar of length l and the vertical.