Central Forces

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Leave all answers in terms of the variables given or any constants necessary; the gravitational constant will be used many times.

1. Patrick on a planet of radius R and mass M experiences a gravitational force of g on its surface. Find the ratio of the gravitational force Patrick would be experience if he was a distance r below the surface with $r < R$. Assume the planet has uniform mass density.

   Remark. *Remember that only the mass within the radius r actually matters because we may consider the mass outside to be made of concentric hallow spherical shells with no gravitational force on the inside.*

2. Patrick is a small mass m that is a distance d away from an infinite line of asteroids with linear density λ. Find the gravitational force which Patrick will feel as a result of the asteroids.

   Remark. *The physics behinds this problem is not very interesting or deep. But, the integration technique is important and everyone should figure out how to do this problem.*

3. We have two definitions of gravitational potential energy. Show that they are equivalent for a small distance away from Earth.

4. Suppose that Patrick is infinitely far away from his breakfast. Since Patrick eats a lot, he has mass m and his breakfast has mass M. When the two masses are a distance r apart. What are their relative speeds?

   Remark. *Conserve energy and momentum. Use two different velocities for the two masses. I also found a solution with the Lagrangian. However, I'm not sure how happy or sad I will be if you present me with a Lagrangian solution.*

5. Patrick builds a very tall rod that reaches up into space. The rod has uniform mass density λ and will be planted into the ground and travels with the same angular velocity ω as the Earth mass m. Find how far along the rod r would the rod most likely break.

   Remark. *The point where the rod is most likely to break is the point where tension is maximum. The tension is caused by the difference in the gravitational force and the centrifugal force felt by a small part of the rod.*

6. ***Challenge*** Below is the equation for the orbit of a particle moving under a central force f

$$\frac{d^2u}{d\theta^2} + u = \frac{-1}{ml^2u^2} f(u^{-1}).$$

   We have used $u = \frac{1}{r}$. Consider Patrick to be moving with a trajectory of $r^2 = R^2\cos(2\theta)$. Find the force $f(r)$ that allows for this trajectory.