Central Forces Part 2

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Leave all answers in terms of the variables given or any constants necessary.

1. One of Jupiter's moons, Europa, is said to have water deep underneath a surface layer of ice. Patrick takes a very long straw and sucks out a bit of water mass m from the center of Europa which has a radius of r up to the surface. Assume that Europa has uniform density ρ, fine much work did Patrick do to suck up the water.

2. A planet has a radius r and uniform density ρ is made of cheese. Patrick the maggot eats away a spherical cavity of the planet that has its center a distance d from the center of the planet. Find the gravitational field strength inside the cavity.

   Remark. *There is actually a Sardinian cheese called Casu marzu, otherwise known as maggot cheese because flies lay their eggs in the cheese and people eat this stuff.*

3. A neutron star is one of the densest objects in the universe. The rotational speed of the star actually prevents gravity from collapsing the star in on itself. If a neutron star has a density ρ, find the period in which the rotation of the neutron star would have to have to not collapse.

4. Patrick is not convinced that the planets in our solar system orbit the sun in planar orbit. Explain qualitatively (without Kepler's laws) what might have happened in the formation of the solar system to cause the planets to have planar orbits.

   Remark. *Always think about angular momentum! It is more of your friend than all of your other friends combined.*

5. Gauss' Law for gravitation states
$$\int \mathbf{g} \cdot dA = -4\pi G \sum M$$
   Where the LHS of the equation is the integral of the gravitational field $\mathbf{g}$ over a surface with an outward pointing normal dA. The RHS is the sum of the masses enclosed in the surface. Patrick is a distance ℓ aways from a very massive slab of mass. The slab has area A and height h and density ρ. Give an equation for the time it would take for Patrick to travel the separation distance ℓ. Notice that he is accelerated by the gravitational force of the planet.

6. ***Challenge*** This is an absolutely crazy problem! Consider a small block mass m_1 which lies between a very massive block of mass $m_2 = 16 \cdot 100^N$, for N is an integer, and a wall. The large block initially moves with some velocity u_0 towards the stationary small block. What ensues is that the smaller block will hit the wall and bounce back and hit the massive block. After how many bounces will the massive block, m_2 start moving backwards.

   Remark. *Take the velocities of both blocks at some n and $n+1$ bounce. A matrix raised to some power is not easy to do in most cases so I suggest you diagonalize the matrix that acts on the nth velocities which gives new $n+1$ velocities. It turns out that for some given N the number of bounces is is the number formed by the first N digits of π.*