Electric Potential Problems

M. Yu

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1 Electric Potential

The electric field is a conservative field and thus it is path independent. This means that for two points in space, the line integral from P_1 to P_2 need not have a specified path of integration. We thus write the scalar quantity of potential as

$$\phi_{21} = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{s}. \tag{1}$$

The electric field is the negative gradient of potential $\mathbf{E} = -\nabla\phi$. Consider Gauss' Law in which we will take the divergence of the electric field to get

$$\begin{aligned} \nabla \cdot \mathbf{E} &= -\nabla \cdot \nabla\phi \\ \nabla \cdot \mathbf{E} &= -\nabla^2\phi \\ &. \end{aligned} \tag{2}$$

The last operator on the right hand side is known as the Laplacian. We will see later that a function that satisfies Laplace's equation $\nabla^2\psi = 0$ has a lot of significance, and is known as a harmonic.

2 Problems

1. Consider a rod of length L and charge q that is uniformly distributed and has its center at the origin and positioned on the z-axis. Thus the top end is at $z = \frac{L}{2}$ and the bottom end is at $z = \frac{-L}{2}$. Determine the potential at some point z on the z axis that is not part of the rod.

2. We introduce the idea of capacitance. We can define capacitance $C = \frac{Q}{V}$. To find the energy we integrate $V dq$. If two parallel plates both with charge Q on them are spaced d apart, determine the force needed to keep the plates separated.

3. Consider the first problem but replace the rod with a dipole with charges $+q$ and $-q$ at the two ends. Determine the potential of an arbitrary point a distance $r >> \frac{L}{2}$ from the center of the dipole. Work under the approximation $\frac{1}{1 \pm \epsilon} \approx 1 \mp \epsilon$. Now take the gradient in polar of the potential to get the electric field.