# Electric Fields

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**Disclaimer: The first two problems were taken from Morin's book on EM**

I would like to share an integration technique which may be very useful in evaluating integrals. In most cases you can do the problems by integration by parts, but it would be fun to see when this new technique can be applied to these problems, too. Furthermore, this tool will serve you well when you learn quantum. Consider the following integral

$$\int_{-\infty}^{\infty} x^2 e^{\frac{-x^2}{2}} dx. \tag{1}$$

Consider adding a new parameter $\alpha$ on the exponent to get $e^{\frac{-\alpha x^2}{2}}$. Taking a derivative we get $\frac{\partial}{\partial \alpha}\left[e^{\frac{-\alpha x^2}{2}}\right] = \frac{-x^2}{2} e^{\frac{\alpha x^2}{2}}$. But now we see that

$$\int_{-\infty}^{\infty} x^2 e^{\frac{-\alpha x^2}{2}} dx = -2\frac{\partial}{\partial \alpha} \int_{-\infty}^{\infty} e^{\frac{-\alpha x^2}{2}} dx = -2\frac{\partial}{\partial \alpha} \left(\frac{2\pi}{a}\right)^{1/2} = \frac{1}{a}\left(\frac{2\pi}{a}\right)^{1/2}. \tag{2}$$

Letting a= 1 gives us our desired answer to (1). The challenging part of this trick at times is finding the right parameter.

# 1 Problems

1. Consider a hollow cone, with no base and half angle at the vertex $\theta$, which has charge density $\sigma$ on its lateral face and a slant height of $L$. A charge $q$ is located at the vertex of the cone. Find the force which the charge experiences.

2. Consider the electric field due to a charged ring of charge density $\lambda$ of radius $b$ along the axis perpendicular to the plane of the ring. Find the distance $z$ on the axis, such that the electric field is maximized.

   **Remark.** *There is a very neat trick envolving AM-GM that you can used to maximize the function if you do not like taking derivatives, like I don't.*

3. Consider a charge $q$ placed at the vertex of a unit cube. Find the electric flux through a face of the cube using a symmetric method, and also simply using integration. The latter part of this problem was given to me by Mr. Schafer. Hopefully, you see how helpful symmetry can be.