F=ma Review

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Leave all answers in terms of the variables given and any constants necessary. Here are some useful formulas.

$$t_{top} = \frac{v_0 \sin\theta}{g}, \quad y_{max} = \frac{v_0^2 \sin^2\theta}{2}, \quad x_{max} = \frac{v_0^2 \sin 2\theta}{g}[1], \quad F_c = \frac{mv^2}{R}, \quad m\ddot{x} = -kx, \quad \tau = I\alpha$$

$$KE_t = \frac{mv^2}{2} + \frac{I\omega^2}{2}, \quad L = I\omega, \quad \Delta L = \Delta pr, \quad \ddot{\mathbf{z}} = -something\mathbf{z} \quad \Rightarrow \omega = \sqrt{something}$$

$$k_{eff} = \frac{k_1 k_2}{k_1 + k_2}, \quad F = \frac{dp}{dt}, \quad P = Fv, \quad I = I_{cm} + mr^2$$

1. Patrick has 3 vertical springs all with spring constant k that he places in parallel on a flat ground. On top of those springs rests a mass less platform, from which 2 more vertical springs each identical to the first 3 are placed in parallel. Then another mass less platform rests on the 2 veritable springs. A mass m is placed on top. How far down does the spring system depress?

2. Patrick and Matthew each have a hose and they shoot water at each other Suppose that the radius of Matthew's hose is $3r$ in radius and Patrick's hose is only r in radius. If Matthew shoots water at a speed of v, find how fast Patrick would need to shoot water if the two streams of water hit each other and do not move.

   Remark. *Think of this problem as one of those battles where two sides shoot fire at each other and neither gets the advantage.*

3. Patrick has horrible coordination and thus always bunts in a baseball game. Matthew pitches a baseball of mass m with velocity v and the ball hits the bat which has length ℓ and mass m perpendicularly. Patrick holds the bat by one end, but is too weak to keep the bat in the air after the ball hits it. Where should the ball hit the bat if the bat and ball both move off with the same velocity after the collision?

   Remark. *The bat will be spinning and translating after the collision. The translational velocity of the bat should equal the speed of the ball after the collision. Conserve momentum, energy, and angular momentum.*

4. Patrick has a chain with linear density λ spread out in a straight line on a frictionless table. He takes his hand and collects the chain so that it forms heap in his hand. If his hand moves at a speed of v at all times, find the force as a function of x, the distance his hand moves. (Everyone must be able to do this problem)

5. Patrick who has mass m designs a jet pack which has a P watt engine. Patrick knows that he can reach a velocity v if he flies to a height of h above the ground. If now he needs to carry Matthew to the same height, how fast will they be going when they reach height h above the ground.

6. Patrick does not play sports nor does he have friends to play with because he lives in the middle of the woods. Matthew decides to teach Patrick to play tether ball. The ball of mass m is hit so that the string makes an angle ϕ with the vertical, tension in the rope is T and the rope is of length ℓ. Find the period of this ball so that Patrick will know when to hit it again.

[1] I expect everyone to be able to beat any 2-d motion problem down to the ground until it weeps in submission, like the Romans at the hands of he Visigoths.